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THEORETICAL STUDY OF HYDRODYNAMICS AND HEAT TRANSFER BY FREE CONVECTION OF NON-NEWTONIAN FLUIDS NEAR A COOLED SURFACE WITH REGARD FOR VARIABLE VISCOSITY

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We have made a theoretical study of the hydrodynamics and heat transfer of non-Newtonian fluids near a cooled isothermal surface by laminar free convection with regard for the change in the fluid viscosity with temperature. A power rheological model has been used. The solutions to the systems of differential equations for the boundary layer have been obtained numerically. It has been shown that the strongest influence in the considered kinds of convection is produced by the relative viscosity. Moreover, of great importance is the nonlinearity index of the medium. With increasing rheological parameter the influence of variable viscosity decreases. Criteria equations for calculating the local and mean Nusselt numbers and friction coefficients have been obtained.

The dependence of the wall shear stress on the shear deformation rate for high-viscosity fluids is nonlinear, as a rule, i.e., they are non-Newtonian media. Temperature strongly influences viscosity, while the other physical properties of dropping liquids depend weakly on temperature and practically do not affect the heat transfer [1]. In [2], the heat transfer of non-Newtonian fluids by free convection with regard for the variable viscosity near surfaces heated with respect to the fluid was investigated. In [3, 4], the hydrodynamics and free convection heat transfer of pseudoplastic and dilatant fluids in the case of constant viscosity were investigated. In [5], the integral method was used to obtain solutions as well as experimental data for an isothermal horizontal cylinder exchanging heat with a fluid obeying the power law. In [6], the heat transfer to non-Newtonian fluids from a heated isothermal vertical plate immersed in 0.5% and 1% aqueous solutions of carboxypolymethylene ("carbopol"), for which $0.7 < n < 1$, were studied experimentally. The free laminar convection of Newtonian fluids with regard for the variable viscosity near both heated and cooled surfaces was investigated in [7]. There is an acute shortage of works on the heat transfer near cooled rheologically complex media, while such investigations are demanded by practice.

In the present work, we have investigated the local heat transfer and friction by free convection of non-Newtonian fluids near an isothermal surface cooled with respect to the medium flowing past it. We used the power rheological model [2]

$$\tau_x = (\mu\dot{\gamma})^n = \mu_{\text{fl}}^n \left(\frac{\mu}{\mu_{\text{fl}}}\dot{\gamma}\right)^n , \quad (1)$$

where μ_{fl} and μ are the viscosities of the fluids outside the boundary layer (constant) and at an arbitrary point of the boundary layer, respectively.

With the use of this model the system of differential equations of free convection heat transfer in the Boussinesq approximation (the equations hold for both the vertical plate and the horizontal cylinder) with regard for the variable viscosity takes on the following form [5]:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta\,(t - t_{\text{fl}}) + \frac{K}{\rho}\frac{\partial}{\partial y}\left(\left[\frac{\mu}{\mu_{\text{fl}}}\right]^n \frac{\partial u}{\partial y}\left|\frac{\partial u}{\partial y}\right|^{n-1}\right), \quad (2)$$

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$$u\frac{\partial t}{\partial x}+v\frac{\partial t}{\partial y}=a\frac{\partial^2 t}{\partial y^2}\,. \tag{3}$$

The boundary conditions are

$$y=0:\;\; u=v=0 \;\;\text{and}\;\; t=t_{\mathrm{w}}\,;\;\; y\to\infty:\;\; u=0 \;\;\text{and}\;\; t=t_{\mathrm{fl}}\,. \tag{4}$$

In view of the Grashof, Prandtl, and Rayleigh criteria taken according to Acagi [2] Eqs. (2)–(4) were transformed by means of the self-simulated variables for the vertical plate and the horizontal cylinder. In both cases, the transformations yield the same system of differential equation

$$\mathrm{Gr}^{*\frac{2-n}{n}}\,\mathrm{Ra}^{*\frac{2n+2}{n(3n+1)}}\left[\left(\frac{2n+1}{n}\right)f(\eta)\,f''(\eta)-\left(\frac{n+1}{n}\right)(f''(\eta))^2\right]+\theta(\eta)+\left[\left(\frac{\mu}{\mu_{\mathrm{fl}}}f''(\eta)\right)^{n}\right]'=0\,, \tag{5}$$

$$\theta''(\eta)+\left(\frac{2n+1}{n}\right)f(\eta)\,\theta'(\eta)=0\,, \tag{6}$$

$$\eta=0:\;\; f(\eta)=0\,,\;\; f'(\eta)=0\,,\;\; \theta(\eta)=1\,;\;\; \eta\to\infty:\;\; f''(\eta)=0\,,\;\; \theta(\eta)=0\,. \tag{7}$$

At $\mathrm{Pr}^*\to\infty$ Eq. (5) is simplified:

$$\theta(\eta)+\left[\left(\frac{\mu}{\mu_{\mathrm{fl}}}f''(\eta)\right)^{n}\right]'=0\,. \tag{8}$$

Here it is necessary to express the variable-viscosity-containing term in terms of constant parameters and the similarity variable. Christiansen [8] transformed expression (1) as follows:

$$\tau=k\left[\dot{\gamma}\exp\left(\frac{E_v}{rT}\right)\right]^{n}\,. \tag{9}$$

Metzner [9] with the aid of Eq. (9) explains the heat transfer of non-Newtonian fluids. Calculations on the basis of this equation from experimental data of a conventional heat exchanger for the majority of non-Newtonian fluids at normal temperatures yield a good coincidence. From comparison of (1) and (9) we have

$$\mu=k^{1/n}\exp\left(\frac{E_v}{rT}\right). \tag{10}$$

Fluid viscosities near the surface μ_{w} and at a distance from it μ_{fl} are constant quantities. With the use of Eq. (10) the ratio of the fluid viscosity μ at an arbitrary point of the boundary layer at temperature T to the fluid viscosity at a distance from the surface μ_{fl} at temperature T_{fl} can be expressed in terms of the ratio of viscosities μ_{fl} and μ_{w}, respectively, at temperatures T_{fl} and T_{w}:

$$\frac{\mu}{\mu_{\mathrm{fl}}}=\left(\frac{\mu_{\mathrm{w}}}{\mu_{\mathrm{fl}}}\right)^{\frac{T_{\mathrm{w}}}{T_{\mathrm{w}}-T_{\mathrm{fl}}}\,\frac{T-T_{\mathrm{fl}}}{T}}=\left(\frac{\mu_{\mathrm{fl}}}{\mu_{\mathrm{w}}}\right)^{\frac{(1+\grave{T})\theta}{1+\grave{T}\theta}}\,.$$

The solutions to the systems of differential equations have been obtained by the fourth-order Runge–Kutta method in the range of change in the parameters: $\bar{\mu}$ = $\mu_{\mathrm{fl}}/\mu_{\mathrm{w}}$ = 0.005–1; n = 0.1–2.5; $\grave{T}$ = –0.5; $\mathrm{Pr}^*\to\infty$. To find the velocity and temperature gradients on the wall, the shooting method was used.

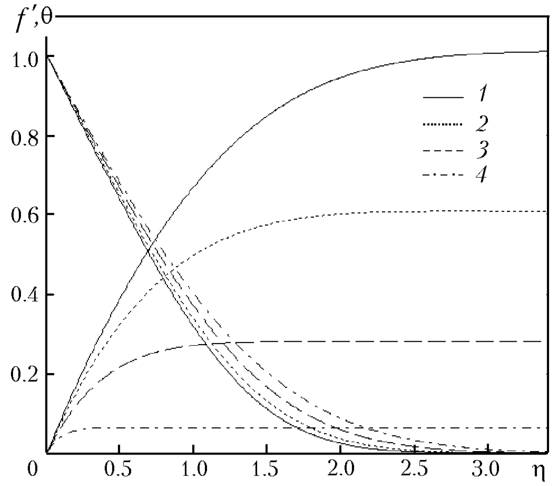


Fig. 1. Influence of the nonlinearity index of the medium on the velocity (f') and temperature (θ) profiles in the case of constant physical properties of the fluid: 1) n = 2.5; 2) 1.25; 3) 0.5; 4) 0.1; f', ascending, θ, descending profiles.

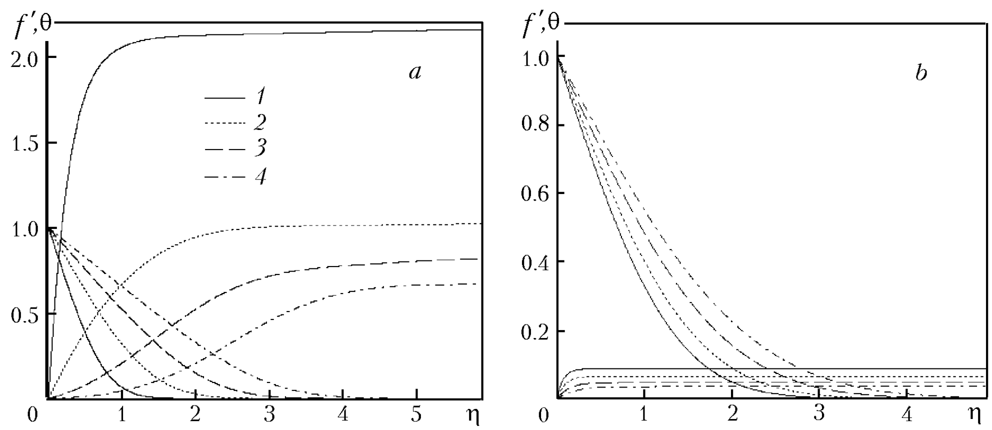


Fig. 2. Influence of variable viscosity on the velocity (f') and temperature (θ) profiles of dilatant (a) — n = 2.5 and pseudoplastic (b) — n = 0.1 fluids: 1) $\bar{\mu}$ = 10; 2) 1; 3) 0.1; 4) 0.01; f', ascending, θ, descending profiles.

Analysis of the obtained solutions at $\bar{\mu}$ = 1 has revealed the influence of rheology on the hydrodynamics and heat transfer. The velocity and temperature profiles for this case at various values of the structural viscosity coefficient are given in Fig. 1. In general, the influence of n on the dynamic parameters of the boundary layer is much stronger than on the thermal ones.

With decreasing rheological parameter n the thickness of the dynamic boundary layer decreases. The thickness of the thermal boundary layer increases with decreasing n, but the scale of these changes is small compared to the changes in the dynamic boundary layer thickness, which agrees with the results of the analysis of the differential equation (6).

With decreasing n the velocity profiles become less filled (more sloping), but the shape of the curves remains practically unaltered, and the maximum velocity thereby decreases. The influence of the index of non-Newtonian behavior of the medium on the temperature profiles is not as strong as on the velocity profiles. With increasing n the velocity and temperature gradients decrease throughout the layer thickness. The results obtained agree with the results of the solutions [2–4, 7] for constant physical properties of the fluid.

Figure 2a shows the influence of the variable fluid viscosity on the velocity and temperature profiles of dilatant media (n = 2.5). Cooling of the fluid near the wall leads to a deformation of the velocity and temperature profiles towards a decrease in their gradients on the wall, and its heating causes an increase in the gradients on the wall with

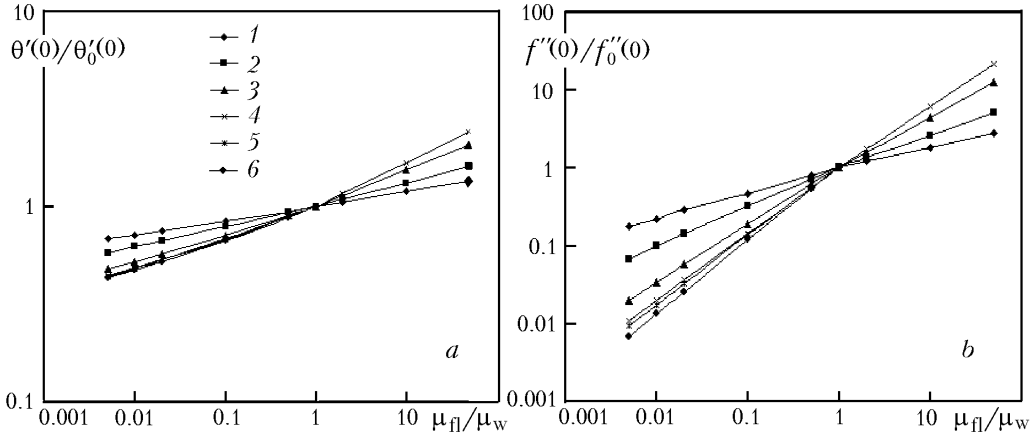


Fig. 3. Relative temperature (a) and velocity (b) gradients on the wall versus variable viscosity at various n ($\mu_{fl}/\mu_w < 1$, data of the authors; $\mu_{fl}/\mu_w > 1$, Acagi's data [2]): 1) n = 0.1; 2) 0.2; 3) 0.5; 4) 1; 5) 1.25; 6) 2.5.

the physical properties of the fluid remaining unaltered. Cooling leads to an increase in the thickness of the thermal boundary layer, and heating causes its decrease. With decreasing $\bar{\mu}$ the maximum velocity decreases. Changes in this parameter have a stronger effect on the dynamic boundary layer than on the thermal one. Analysis of the results obtained has shown that the relative viscosity produces a stronger effect on the thermal parameters than the nonlinearity coefficient n does, and on the dynamic parameters — vice versa.

With increasing $\bar{\mu}$ the thermal boundary layer thickness decreases, and the thickness of the dynamic boundary layer is practically independent of the variable viscosity. This is explained by the fact that viscosity and temperature vary within the limits of the thermal boundary layer. For high-viscosity fluids the latter is much thinner than the dynamic boundary layer, and since the longitudinal velocities of different fluid layers tend to equalize at a distance from the wall, the difference in velocity profiles for various values of the variable viscosity manages to level off in the remaining part of the dynamic boundary layer.

With decreasing $\bar{\mu}$ the temperature profiles become more sloping. The smaller this parameter, the higher the temperature curve. The velocity profiles at $\bar{\mu} < 1$ gradually deform, an inflection point appears on them, the curves become increasingly S-shaped, and the maximum longitudinal velocity thereby decreases. Such a velocity profile is also observed in Newtonian fluids (n = 1) in the same range of change in the relative viscosity [7], which points to the presence of a slow-moving layer near a cooled surface, but in dilatant fluids the "creeping" motion effect is much stronger.

With decreasing $\bar{\mu}$ the velocity and temperature gradients on the wall decrease. A more pronounced S-shaped velocity profile decreases the stability of the laminar flow of dilatant fluids compared to Newtonian fluids, which may lead to the separation of the boundary layer or precipitate the transition to a turbulent flow. For an individual dilatant medium the same is observed when the velocity gradient tends to zero with decreasing parameter $\bar{\mu}$.

At small values of the flow index n the influence of $\bar{\mu}$ decreases (Fig. 2b), and the S-shaped deflection of the velocity profile becomes unnoticeable. In other respects for pseudoplastics ($n < 1$) the foregoing holds for dilatant fluids ($n > 1$).

Analysis of the results of the solutions has shown that the influence of the variable viscosity on the velocity and temperature gradients on the wall is fairly reliably estimated by the parameter $\bar{\mu}$. These gradients with respect to the quantities for constant properties of the fluid are well approximated by the following relations (Fig. 3):

$$\theta'(0)/\theta_0'(0) = \mathrm{Nu}/\mathrm{Nu}_0 = \bar{\mu}^{m} , \tag{11}$$

$$f''(0)/f_0''(0) = \bar{\mu}^{k} , \quad C_f/C_{f0} = \bar{\mu}^{n(k-1)} . \tag{12}$$

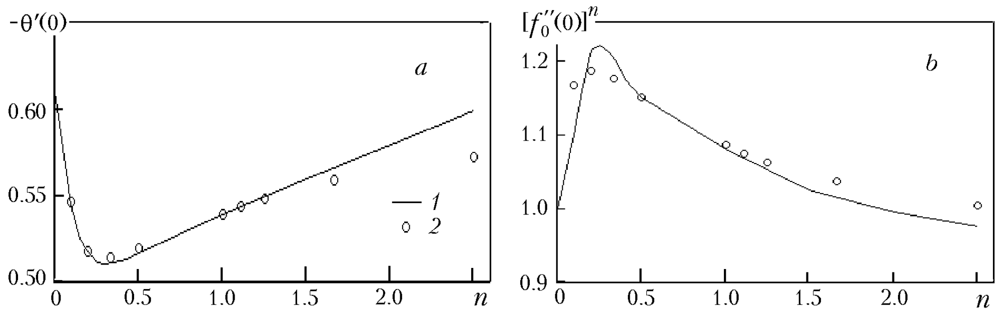


Fig. 4. Temperature gradient — θ′(0) (a) and velocity gradient of *n*th degree $[f''(0)]^n$ (b) versus the flow index *n* (physical properties of the fluid are constant). 1) Acrivos's data [4]; 2) our data.

The indices *m* and *k* are functions of the flow index *n* and are generalized with an error of no more than ± 1.2% by the relations

$$m = 0.174 - \frac{0.019}{n + 0.084}, \tag{13}$$

$$k = 1.016 - \frac{0.193}{n + 0.180}, \quad n\,(k - 1) = 0.016n - \frac{0.193n}{n + 0.180}. \tag{14}$$

Using relations (11)–(14) and taking into account the values of the parameters $f''(0)$ and $\theta'(0)$, we have obtained criteria equations for the local and average heat transfer and friction. In the case of the vertical plate (mean errors are, respectively, ±2, ±0.8, ±0.8, and ±0.5%)

$$\mathrm{Nu}_x = \left(0.530 - \frac{0.639}{n + 26.949}\right) \mathrm{Ra}_x^{*\,1/(3n+1)}\, \bar{\mu}^{\,m}, \tag{15}$$

$$\overline{\mathrm{Nu}} = \left(0.887 - \frac{0.682}{n + 2.099}\right) \mathrm{Ra}^{*\,1/(3n+1)}\, \bar{\mu}^{\,m}, \tag{16}$$

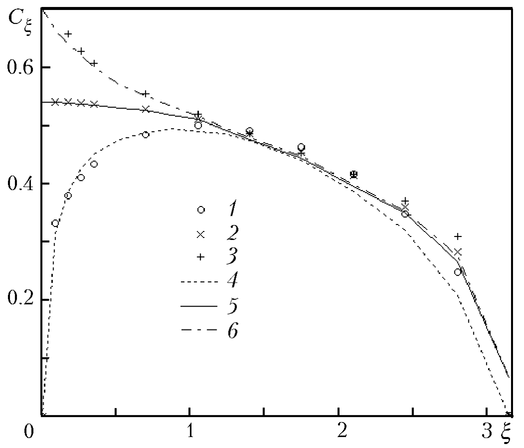
$$C_{fx} = \left(\frac{6.189}{n + 7.807} + 1.628\right) \mathrm{Ra}_x^{*\,-1/(3n+1)}\, \bar{\mu}^{\,n(k-1)}, \tag{17}$$

$$\overline{C}_f = \left(\frac{2.772}{n + 0.016} + 6.560\right) \mathrm{Ra}^{*\,-1/(3n+1)}\, \bar{\mu}^{\,n(k-1)}. \tag{18}$$

Analogous expressions for the free convection of the non-Newtonian fluid near the horizontal cylinder (mean errors are, respectively, ±0.3, ±0.9, ±0.06, and ±0.1%) are

$$\mathrm{Nu}_x = \left(0.797 - \frac{0.211}{n + 1.405}\right) \mathrm{Ra}^{*\,1/(3n+1)}\, g\,(\xi)\, \bar{\mu}^{\,m}, \tag{19}$$

$$\overline{\mathrm{Nu}} = \left(0.606 - \frac{0.549}{n + 2.178}\right) \mathrm{Ra}^{*\,1/(3n+1)}\, \bar{\mu}^{\,m}, \tag{20}$$



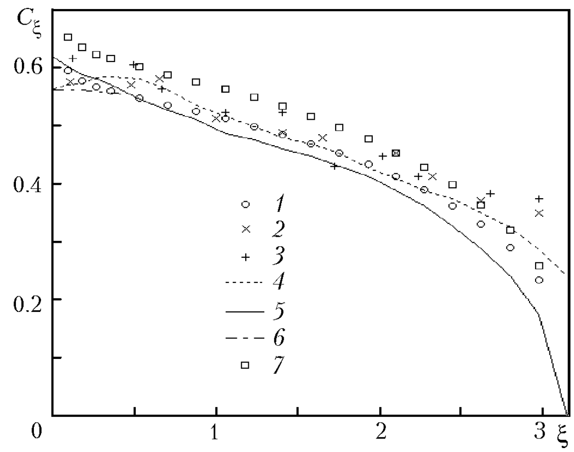


Fig. 5. Change in the local heat transfer coefficient along the surface of the horizontal isothermal cylinder: 1) n = 0.5; 2) 1; 3) 1.5 (our data); 4) n = 0.5; 5) 1.0; 6) 1.5 (Acrivos's data [4]).

Fig. 6. Change in the local heat transfer coefficient along the surface of the horizontal isothermal cylinder [5] (working fluid — aqueous solution of saccharose with a 38% content of maize starch): 1) our data for n = 1.18, $\bar{\mu}$ = 1; 2) Δt = 2.2$^{\text{o}}$C; 3) Δt = 8.9$^{\text{o}}$C; 4) experimental data for Newtonian fluids; 5) solution by the integral method for the case of a constant rate of heat flow on the surface (n = 1.18); 6) solution for Newtonian fluids obtained by the integral method; 7) Acagi's data [2] for n = 1.18, $\bar{\mu}$ = 1.5.

$$C_{fx} = \left(\frac{1.144}{n+0.516}+0.729\right)\mathrm{Ra}^{*^{-1/(3n+1)}}\left(s\left(\xi\right)\right)^{n}\left(g\left(\xi\right)\right)^{2n}\bar{\mu}^{\,n(k-1)}\,, \qquad (21)$$

$$\bar{C}_f = \left(\frac{1.160}{n+0.467}+0.745\right)\mathrm{Ra}^{*^{-1/(3n+1)}}\bar{\mu}^{\,n(k-1)}\,. \qquad (22)$$

Here the functions $s(\xi)$ and $g(\xi)$ take into account the cylinder curvature, and their values have been calculated and tabulated.

Figure 4 shows the dependences of the transfer parameters $-\theta'(0)$ and $[f''(0)]^n$ on the exponent n according to Acrivos's data [4] as well as our data. It is seen that in the region of $0 < n < 1.5$ there is a good agreement between our results and Acrivos's solutions, but at $n > 1.5$ a discrepancy is observed. The dependences of $C_\xi = \mathrm{Nu}/\mathrm{Gr}^{*1/(2n+2)}\,\mathrm{Pr}^{*n/(3n+1)}$ on the angle ξ for the horizontal isothermal cylinder obtained theoretically as in [1] at n = 0.5, 1.0, and 1.5 are given in Fig. 5, where $\mathrm{Nu} = \alpha R/\lambda$.

In [5], good agreement between the self-similar solution, the solutions constructed by the integral method, and the experimental data was noted. Figure 6 gives the curves of the local Nusselt number as a function of angle ξ for the isothermal regime of a heated surface [5], and for comparison the same figure presents the results obtained in the present paper for an analogous value of the rheological parameter n = 1.18, but at constant physical properties of the fluid. It made sense to present these results due to the fact that the temperature head in the considered experiments is too small for the viscosity of the substance used to change markedly. For comparison Fig. 6 also gives the Acagi solutions [2] for n = 1.18 and $\bar{\mu}$ = 1.5, where the decrease in the fluid viscosity near the surface upon heating is taken into account. According to the experimental data of [5], the averaged Nusselt numbers turned out to be 5–10% smaller than the theoretical values of the present study.

In the literature there is lack of experimental data and theoretical studies on free convection of non-Newtonian fluids near a cooled surface with regard for the variable viscosity. In [7], such studies (theoretical and experimental)

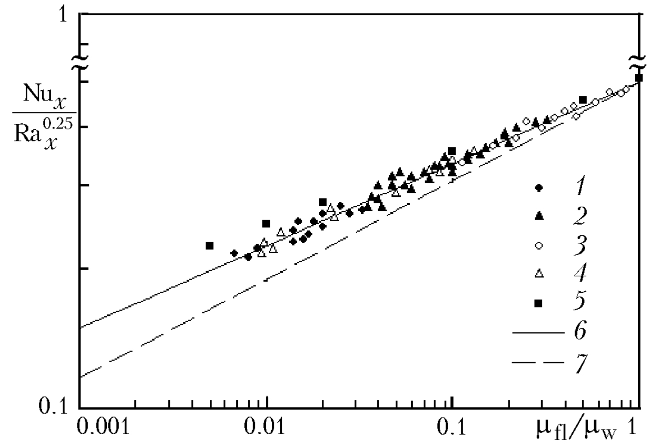


Fig. 7. Comparison of the results obtained by different researchers on local heat transfer (n = 1) with regard for the influence of variable viscosity: 1) [10]; 2) [11]; 3) oil [7]; 4) fuel oil [7]; 5) [12]; 6) our data for n = 1; 7) μ = 1 [7].

for Newtonian fluids were reported. For the above case, the following equations of local heat transfer and friction by free convection near a vertical isothermal wall were obtained there:

$$\mathrm{Nu}_x = 0.503\left(\frac{1}{1+\mathrm{Pr}_x^{-0.5}}\right)^{0.25}\mathrm{Ra}_x^{0.25}\,\bar{\mu}^{0.17}\,, \quad C_{fx} = 2.08\,\mathrm{Ra}_x^{-0.25}\,\mathrm{Pr}_x^{0.013}\,\bar{\mu}^{-0.15}\,.$$

These relations have been confirmed by many experimental and theoretical studies [10–12]. In the present work, the generalized similarity criteria at n = 1 are converted to the values of Newtonian fluids, i.e., they take on the traditional form. According to our data, for the considered case of free convection of a Newtonian fluid near a vertical cooled isothermal wall (at n = 1) the local Nusselt number and the friction coefficient are calculated by the formulas

$$\mathrm{Nu}_x = 0.507\,\mathrm{Ra}_x^{0.25}\,\bar{\mu}^{0.156}\,, \quad C_{fx} = 2.331\,\mathrm{Ra}_x^{-0.25}\,\bar{\mu}^{-0.148}\,.$$

Figure 7 presents the results of the solutions obtained by us for comparison to the experimental data and numerical solutions obtained by different researchers for local heat transfer.

CONCLUSIONS

1. The influence of variable viscosity of a fluid on the relative heat transfer and friction depends only on $\bar{\mu}$. The degree of influence of this parameter on the heat transfer and friction is different and depends on the thermal flow direction and the nonlinearity index of the fluid (rheological parameter n).

2. The influence of variable viscosity on the hydrodynamics is much stronger than on the heat transfer. The same holds for the rheological parameter n.

3. Upon cooling of the fluid near the surface the laminar flow stability decreases compared to the isothermal flow. With decreasing $\bar{\mu}$ and decreasing degree of dilatancy this trend grows.

4. Criteria equations have been obtained for calculating local and mean Nusselt numbers and friction coefficients under laminar free convection near a cooled vertical plate (expressions (15)–(18)) and a cooled horizontal cylinder (expressions (19)–(22)). They can be recommended for engineering calculations of local and mean heat transfer and friction coefficients.

5. Our solutions agree with the experimental and theoretical data obtained by other authors.

NOTATION

a, thermal diffusivity, m^2/sec; c, isobaric specific heat capacity, J/(kg·K); C_f, friction coefficient; E_v, activation energy of one mole, J/mole; $f(\eta)$, dimensionless stream function; g, acceleration of gravity, m/sec^2; $g(\xi) = [((3n+1)/n)\times \times^{-n/(3n+1)} \sin^{1/(2n+1)}\xi]/\left(\int_0^{\xi}\sin^{1/(2n+1)}\xi d\xi\right)^{n/(3n+1)}$, function that, along with the function $s(\xi)$, takes into account the cylinder curvature and permits application of the equations for a vertical plate to a horizontal cylinder; $\mathrm{Gr}_x^* = \rho^{2/(2-n)} g\beta(T_w - T_{fl}) x^{(2+n)/(2-n)}\mu_{fl}^{-2n/(2-n)}$, generalized Grashof similarity number; K, consistency measure, (Pa·sec)^n; k, constant parameter; L, characteristics size of the body (length of the plate, radius of the cylinder), m; n, structural viscosity index; Nu, Nusselt number; $\mathrm{Pr}_x^* \equiv \rho^{1-n}x^{2-2n}\mu_{fl}^{n}c^{2-n}\lambda^{n-2}$, generalized Prandtl similarity number; R, cylinder radius, m; r, universal gas constant, J/(K·mole); $\mathrm{Ra}^* = \mathrm{Gr}^*\,\mathrm{Pr}*^{n/(2-n)}$, generalized Rayleigh similarity number; $s(\xi) = ((3n+1)/n)^{(2n+1)/(3n+1)}\left(\int_0^{\xi}\sin^{1/2n+1}\xi d\xi\right)^{(2n-1)/(3n+1)}$, function having a meaning analogous to $g(\xi)$; T, absolute temperature, K; $T = (T_w - T_{fl})/T_{fl}$, temperature parameter; t, temperature, °C; u, fluid velocity in external flow, m/sec (for the vertical plate $u = \frac{a}{L}\mathrm{Ra}*^{2/(3n+1)}\left(\frac{3n+1}{n}\frac{x}{L}\right)^{(n+1)/(3n+1)} f'(\eta)$, for the horizontal cylinder $u = 2^{-2(2n+1)/(3n+1)}\frac{a}{R}\mathrm{Ra}*^{2/(3n+1)}s(\xi)$ $g(\xi)f'(\eta)$); v, fluid velocity perpendicular to the surface being flown; x, longitudinal coordinate of the plate (for the cylinder x is an arc coordinate reckoned from the front point), m; y, transverse coordinate in the case of the plate, m; β, isobaric expansion coefficient, 1/K; $\dot{\gamma}$, shear deformation rate; 1/sec; η, similarity variable ($\eta(x, y) = \frac{y}{L}\left(\frac{3n+1}{n}\frac{x}{L}\right)^{-n/(3n+1)}\mathrm{Ra}*^{1/(3n+1)}$ for the vertical plate, $\eta = 2^{-(2n+1)/(3n+1)}\frac{y}{R}\mathrm{Ra}*^{1/(3n+1)}\,g(\xi)$ for the horizontal cylinder); $\theta = (T - T_{fl})/\,(T_w - T_{fl})$, dimensionless temperature; λ, heat conductivity coefficient, W/(m·K); μ, dynamic viscosity, Pa·sec; $\bar{\mu}$, relative viscosity; $\xi = x/R$, angular coordinate with respect to the front point of the cylinder, rad; ρ, fluid density, kg/m^3; τ_x, local shear stress along the wall, Pa. Subscripts: fl, fluid; w, wall; f, near the surface being flown; x, local value; v, for one mole of substance; 0, at constant physical properties of fluid; *, modified similarity criteria.

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